**Chapter 12 Pages 307-313**

**TODAY YOU WILL BE ABLE TO…**

* Distinguish between disjointness and independence
* Determine whether two events are independent
* Apply the multiplication rule for independent events
* Determine whether two events are disjoint
* Apply the general addition rule

**DISJOINTNESS VS. INDEPENDENCE**

If two or more events have no outcomes in common, they are called **disjoint** events.

If two events A and B do not influence each other, and if knowledge that one occurred does not change the probability of the other occurring, the events are said to be **independent** of each other.

**Be careful not to confuse disjointness with independence.**

***Example 1:*** Disjoint vs. Not Disjoint (overlap) \*\*Use a Venn diagram to illustrate\*\*

*Define the following events:*

A = the person chosen is 18 years of age or younger

B = the person is 19 years of age or older

*Select a person at random.*

**If A and B are disjoint, the fact that A occurs means   
that event B cannot occur.**

**The person selected cannot be both “18 or younger”   
AND “19 or older.”**

*Define the following events:*

A = the person has a dog

B = the person has a cat

*Select a person at random.*

**If A and B overlap, both A and B can occur.**

**The person selected can have both a cat and a dog.**

***Example 2:*** Independent vs. Dependent \*\***Cannot** use a Venn diagram to illustrate\*\*

Define the sample space S={red, blue} and the following events related to selecting two beads where the first bead will NOT be replaced before drawing the second bead:

A = the first bead chosen is red

B = the second bead chosen is blue

*Are these events independent?*

**Independence has to do with probabilities of the events rather than just the outcomes that make up the events. If A and B are independent events, then the occurrence of A has no effect on the probability of B.**

What is the probability of selecting a red bead on the first draw?

P(A) =

If a red bead was selected on the first draw, what is the probability of selecting a blue bead on the second draw?

P(B) =

If a blue bead was chosen on the first draw, what is the probability of selecting a blue bead on the second draw?

P(B) =

**DETERMINE WHETHER TWO EVENTS ARE INDEPENDENT**

***Example 3.1:*** Are these events independent or not?

1. Can you taste PTC? PTC is a substance with an unusual property: 70% of people find that it has a bitter taste and the other 30% cannot taste it at all. Define the following events and choose two people at random:

A = the first person can taste PTC

B = the second person can taste PTC

Does the ability of the first person to taste or not taste PTC influence the ability of the second person?

1. Flip a fair coin three times. Is the result of each flip independent of the other flips?
2. Define the following events and deal two cards from a standard deck:

A = the first card dealt is red

B =the second card dealt is red

Are the colors of successive cards dealt from the same deck independent?

**THE MULTIPLICATION RULE FOR INDEPENDENT EVENTS**

Two events are independent if knowing that one occurs does not change the probability that the other occurs. If A and B are independent,

P(A and B) = P(A)P(B)

***Example 3.2:*** Are these events independent or not?

1. Can you taste PTC? PTC is a substance with an unusual property: 70% of people find that it has a bitter taste and the other 30% cannot taste it at all. Define the following events and choose two people at random:

A = the first person can taste PTC

B = the second person can taste PTC

P(both people can taste PTC) =

1. Flip a coin three times:

P(three heads in succession) =

**DETERMINE WHETHER EVENTS ARE DISJOINT**

***Example 4:***

*Given a pair of fair die, define events A and B as possible outcomes of rolling the die…*

A = the sum of the dots is 11

B = a pair is rolled

*Are events A and B Disjoint?*

*Given a pair of fair die, define events C and D as possible outcomes of rolling the die…*

C = the sum of the dots is 12

D = a pair is rolled

*Are events A and B Disjoint?*

***Example 5.1:***

Motor vehicles in the U.S. are classified as either cars or light trucks and as either domestic or imported. In 2010, 76% of new vehicles sold to individuals were domestic, 50% were light trucks, and 43% were domestic light trucks. Choose a vehicle sale at random.

A = the sale chosen involved the sale of a light truck

B = the sale chosen involved the sale of a domestic light truck

**Are these events disjoint?**

**THE GENERAL ADDITION RULE**

Recall that if *A* and *B* are **disjoint** events,

*P*(*A* or *B*) = *P*(*A*) + *P*(*B*)

**The following addition rule works for any two events:**

For any two events *A* and *B*:

***P*(*A* or *B*) = *P*(*A*) + *P*(*B*) – *P*(*A* and *B*)**

***Example 5.2:***

Motor vehicles in the U.S. are classified as either cars or light trucks and as either domestic or imported. In 2010, 76% of new vehicles sold to individuals were domestic, 50% were light trucks, and 43% were domestic light trucks. Choose a vehicle sale at random.

A = the sale chosen involved the sale of a light truck

B = the sale chosen involved the sale of a domestic light truck

1. What is the probability that the sale chosen is domestic or a light truck?
2. What is the probability that the sale chosen is an imported car?
3. What is the probability that the sale chosen is a domestic car?

**Chapter 12 Pages 314-321**

**TODAY YOU WILL BE ABLE TO…**

* Define conditional probability
* Compute conditional probabilities
* Apply the general multiplication rule
* Describe chance behavior with a tree diagram

The probability we assign to an event can change if we know that some other event has occurred.

***Example 5.3:***

The following table gives the probabilities for a randomly chosen light motor vehicle sold at retail in the U.S.

|  |  |  |  |
| --- | --- | --- | --- |
|  | **Domestic** | **Imported** | **Total** |
| **Light Truck** | 0.43 | 0.07 | 0.5 |
| **Car** | 0.33 | 0.17 |  |
| **Total** |  |  | 1 |

The probabilities in the body of the table (light grey cells) sum to 1 (dark grey) because they describe all vehicles sold.

The **Total** rows and columns, marginal probabilities, are obtained by the addition rule:

P(domestic light truck or imported light truck) =

P(domestic light truck)+P(imported light truck) = 0.43+0.07 = 0.5

**Calculate the following:**

P(domestic light truck or domestic car) =

P(imported light truck or imported car) =

P(domestic car or imported car) =

Suppose you know that the vehicle chosen is imported, i.e., it is one of the 24% in the **Imported** column of the table. What proportion of imported vehicles are light trucks?

P(truck | imported) = = 0.29

The pipe, “|”, means “given the information that”

What is the probability that the vehicle is a truck **given the information that** the vehicle is imported?

**Calculate the following:**

What proportion of domestic vehicles are cars?

P(car | domestic) =

What proportion of cars are imported?

P(imported | car) =

**THE CONDITIONAL PROBABILITY RULE**

When A > 0, the conditional probability of B given A is…

P(B | A) =

**THE GENERAL MULTIPLICATION RULE**

We want to know the probability of both A and B happening. Take the conditional probability formula and solve for P(A and B).

P(A and B) = P(A)P(B | A)

***Note:*** P(A and B) = P(A)P(B) is a special case of the general multiplication rule. If two events are independent, then P(B | A) = P(B) and P(B | A) can be replaced by P(B) in the general multiplication rule.

**TREE DIAGRAMS**

Tree diagrams organize probabilities in complex problems.

About 1% of the American population is allergic to peanuts or tree nuts. Choose three people at random and let the random variable X be the number in the sample who are allergic to peanuts or tree nuts. X = 0, 1, 2, 3.Make a three-stage tree diagram to outline the possible outcomes and respective probabilities to obtain the probability distribution of X.